IE 601: Optimization Theory
Spring 2022

Instructor: **Burak Kocuk** (`burakkocuk@sabanciuniv.edu`)

Lecture Hours: Wednesday 14:40–16:30 (FENS L027), Thursday 16:40–17:30 (FENS 2019).

Office Hours: Wednesday 9:00–10:00 (FENS 2095) or by appointment.

Pre-requisite: IE 501.

Catalog Description: Convex optimization and functional analysis; theory of duality; iterative methods and convergence proofs; interior point methods for linear programming; computational complexity of mathematical programming problems; extensions of linear programming.

Course Topics: This course will cover five main topics:

1. `Background Material:` Basic convex analysis (convex sets, convex functions, regular cones), linear programming (polyhedral representability, duality, Farkas Lemma), convex programming (duality, optimality conditions).

2. `Conic Programming Theory:` Duality, tractable conic programs (second-order cone programming, semidefinite programming), conic representability, other useful cones (exponential cone, power cone).

3. `Conic Programming Applications:` Conic programming relaxations of non-convex optimization problems, applications in robust optimization, portfolio optimization, power systems optimization, statistics/machine learning.

4. `Complexity:` Computational complexity of linear programming, interior-point methods.

5. `Advanced Topics:` Sum-of-squares/moment relaxations for polynomial optimization problems, copositive programming (if time permits).

Software: A Python-based modeling system for convex optimization called `CVXPY` will be used for the examples discussed in class and homework assignments. Please install `CVXPY` from `https://www.cvxpy.org/`.
You are also recommended to install the conic programming solver `MOSEK` and use it in conjunction with `CVXPY`. The details can be found from `https://www.cvxpy.org/install/index.html`.

Reference Books: *Lectures on Modern Convex Optimization*, A. Ben-Tal and A. Nemirovski (SIAM).
Convex Optimization, S. Boyd and L. Vandenberghe (Cambridge University Press).
Numerical Optimization, J. Nocedal and S. Wright (Springer Press).

Grading: Midterm (20%), Final (30%), Homework (30%), Final Project (20%).

Homework: There will be four homework assignments, each featuring some theoretical as well as computer-based questions. Students are allowed to work together as long as they submit their own homework and acknowledge who they have worked with.

Exams: There will be two open-notes (closed-homeworks) exams (tentative dates: **April 13th, June 13th**).

Project: Students are expected to read at least one paper involving a conic programming application and develop a basic implementation.
Students will submit a report explaining their findings by **June 26th**.
Students can choose their own topics in consultation with the instructor, or from the following list:

- `Second-order cone/semidefinite programming relaxations.`

  - Anstreicher, K. M. (2012). On convex relaxations for quadratically constrained quadratic programming. *Mathematical Programming*, 136(2), 233-251.

  - Burer, S., & Anstreicher, K. M. (2013). Second-order-cone constraints for extended trust-region subproblems. *SIAM Journal on Optimization*, 23(1), 432-451.

  - Mitchell, J. E., Pang, J. S., & Yu, B. (2014). Convex quadratic relaxations of nonconvex quadratically constrained quadratic programs. *Optimization Methods and Software*, 29(1), 120-136.

  - Kim, S., & Kojima, M. (2003). Exact solutions of some nonconvex quadratic optimization problems via SDP and SOCP relaxations. *Computational Optimization and Applications*, 26(2), 143-154.

- `Sparsity exploitation in semidefinite programming.`

  - Fukuda, M., Kojima, M., Murota, K., & Nakata, K. (2001). Exploiting sparsity in semidefinite programming via matrix completion I: General framework. *SIAM Journal on Optimization*, 11(3), 647-674.

  - Waki, H., Kim, S., Kojima, M., & Muramatsu, M. (2006). Sums of squares and semidefinite program relaxations for polynomial optimization problems with structured sparsity, *SIAM Journal on Optimization*, 17(1), 218-242.

- `Scaled diagonally dominant restrictions in semidefinite programming.`

  - Ahmadi, A. A., & Majumdar, A. (2017). DSOS and SDSOS optimization: more tractable alternatives to sum of squares and semidefinite optimization. *SIAM Journal on Applied Algebra and Geometry*, 3(2), 193-230.

- `Applications in discrete optimization.`

  - Rendl, F., Rinaldi, G., & Wiegele, A. (2010). Solving max-cut to optimality by intersecting semidefinite and polyhedral relaxations. *Mathematical Programming*, 121(2),307.

  - De Klerk, E., Pasechnik, D. V., & Sotirov, R. (2008). On semidefinite programming relaxations of the traveling salesman problem. *SIAM Journal on Optimization*, 19(4), 1559-1573.

- De Klerk, E., & Sotirov, R. (2010). Exploiting group symmetry in semidefinite programming relaxations of the quadratic assignment problem. *Mathematical Programming*, 122(2), 225-246.

- Povh, J., & Rendl, F. (2009). Copositive and semidefinite relaxations of the quadratic assignment problem. *Discrete Optimization*, 6(3), 231-241.

- `Copositive programming relaxations.`

  - Burer, S. (2015). A gentle, geometric introduction to copositive optimization. *Mathematical Programming*, 151(1), 89-116.

  - Bomze, I. M. (2015). Copositive relaxation beats Lagrangian dual bounds in quadratically and linearly constrained quadratic optimization problems. *SIAM Journal on Optimization*, 25(3), 1249-1275.

  - Bomze, I. M., Jarre, F., & Rendl, F. (2011). Quadratic factorization heuristics for copositive programming. *Mathematical Programming Computation*, 3(1), 37-57.

- `Applications in power systems.`

  - Kocuk, B., Dey, S. S., & Sun, X. A. (2016). Strong SOCP relaxations for the optimal power flow problem. *Operations Research*, 64(6), 1177-1196.

  - Coffrin, C., Hijazi, H. L., & Van Hentenryck, P. (2016). The QC relaxation: A theoretical and computational study on optimal power flow. *IEEE Transactions on Power Systems*, 31(4), 3008-3018.

  - Molzahn, D. K., & Hiskens, I. A. (2015). Sparsity-exploiting moment-based relaxations of the optimal power flow problem. *IEEE Transactions on Power Systems*, 30(6), 3168-3180.

- `Applications in finance.`

  - Bertsimas, D., Gupta, V., & Paschalidis, I. C. (2012). Inverse optimization: A new perspective on the Black-Litterman model. *Operations Research*, 60(6), 1389-1403.

  - Zhu, S., & Fukushima, M. (2009). Worst-case conditional value-at-risk with application to robust portfolio management. *Operations Research*, 57(5), 1155-1168.

  - Ghaoui, L. E., Oks, M., & Oustry, F. (2003). Worst-case value-at-risk and robust portfolio optimization: A conic programming approach. *Operations Research*, 51(4), 543-556.

- `Applications in statistics/machine learning/signal processing.`

  - Waldspurger, I., d'Aspremont, A., & Mallat, S. (2015). Phase recovery, maxcut and complex semidefinite programming. *Mathematical Programming*, 149(1-2), 47-81.

  - Luo, Z. Q., Ma, W. K., So, A. M. C., Ye, Y., & Zhang, S. (2010). Semidefinite relaxation of quadratic optimization problems. *IEEE Signal Processing Magazine*, 27(3), 20-34.

  - Wang, Z., Zheng, S., Ye, Y., & Boyd, S. (2008). Further relaxations of the semidefinite programming approach to sensor network localization. *SIAM Journal on Optimization*, 19(2), 655-673.

  - Goldfarb, D., & Yin, W. (2005). Second-order cone programming methods for total variation-based image restoration. *SIAM Journal on Scientific Computing*, 27(2),622-645.

  - El Ghaoui, L., & Lebret, H. (1997). Robust solutions to least-squares problems with uncertain data. *SIAM Journal on Matrix Analysis and Applications*, 18(4), 1035-1064.